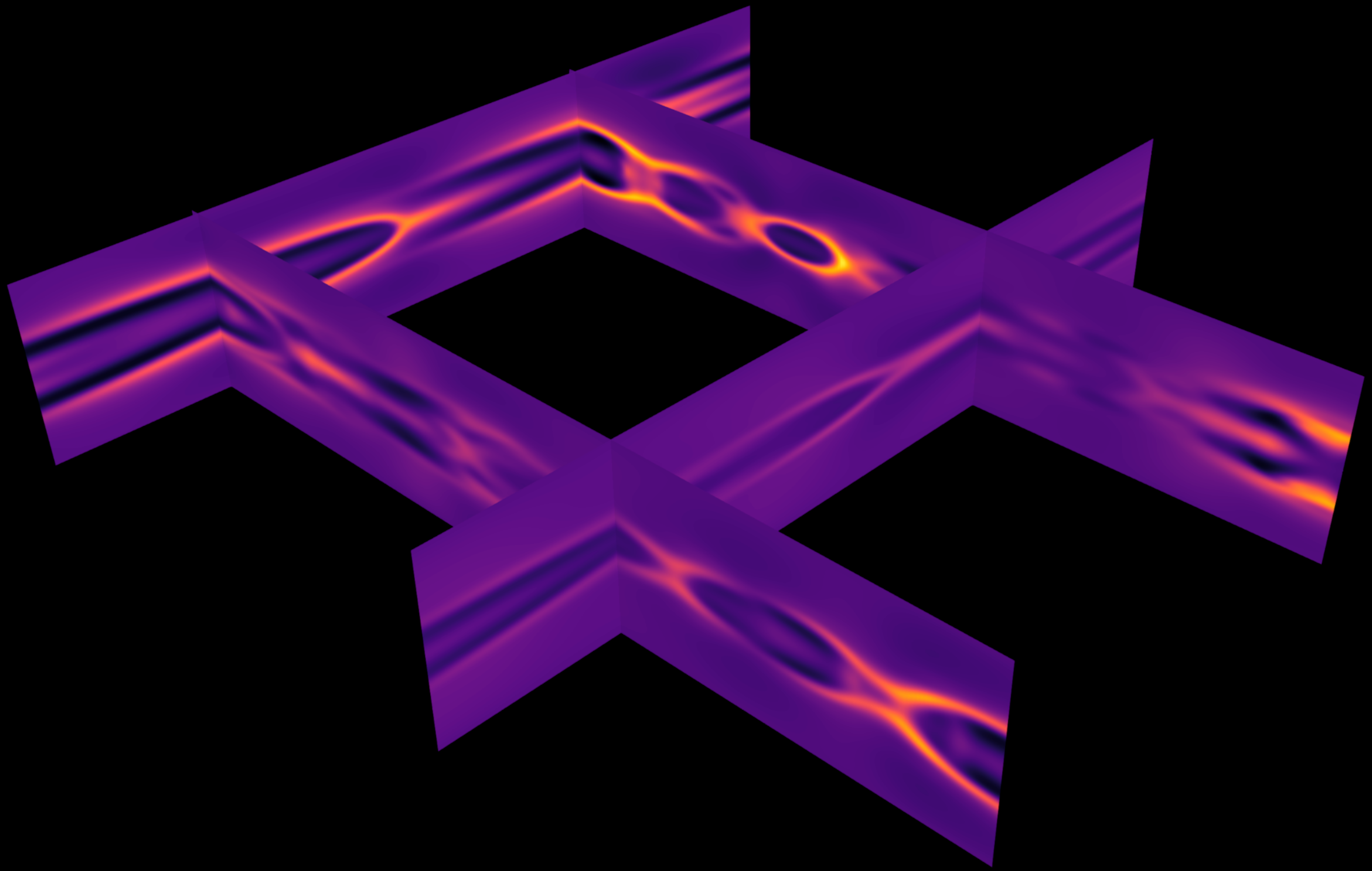
Narwhals and their Blessings on ARCHER2

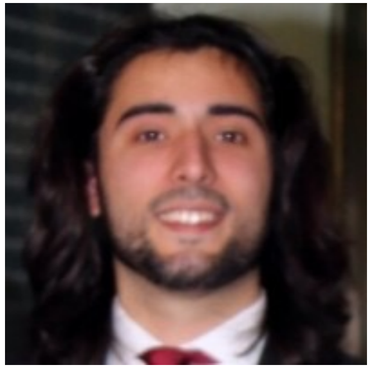
Alexander Morozov

THE UNIVERSITY of EDINBURGH



Team

Martin Lellep



Damiano Capocci

Moritz Linkmann

Funding

UKRI Engineering and Physical Sciences Research Council

archer2

UK TURBULENCE CONSORTIUM

Rebuild Ukraine

Isaac Newton Institute and London Mathematical Society will deliver this new scheme together, designed to support research active Ukrainian mathematicians to come to the UK for short term research visits.

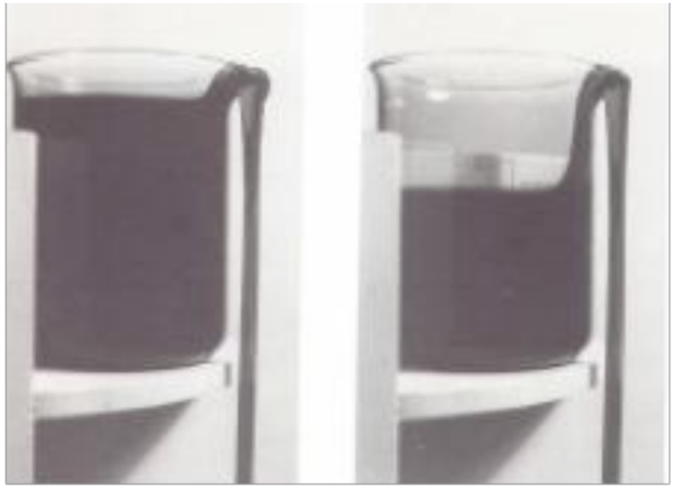
University of Edinburgh Sanctuary Studentships: PhD positions for Ukrainian students in natural sciences, medicine, and veterinary sciences.

sciences.

Unusual flows of polymer solutions

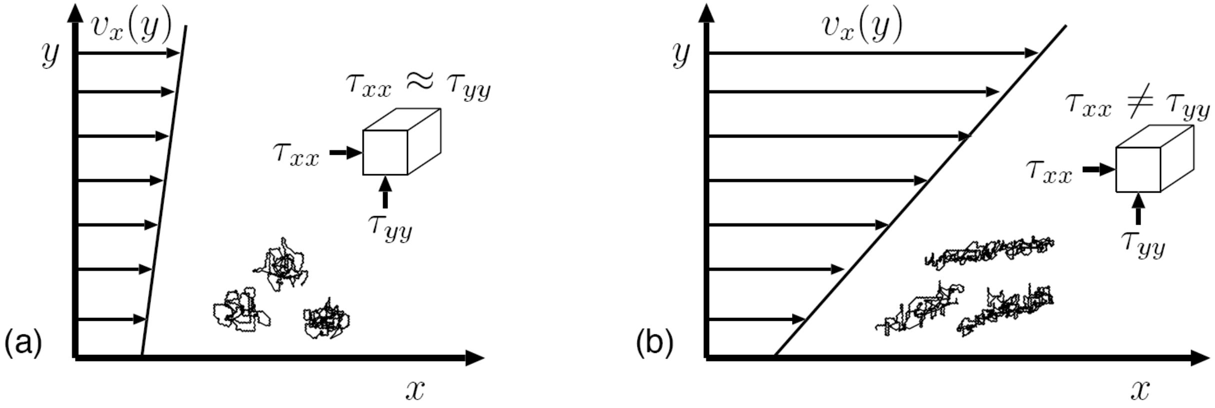
"die swell"

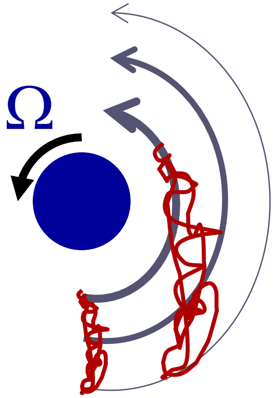
"siphon effect"



"rod climbing"

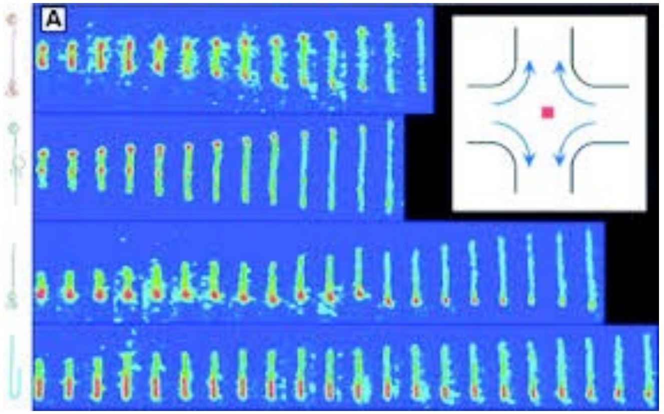
Bird *et al*. Dynamics of Polymeric Liquids (1987)



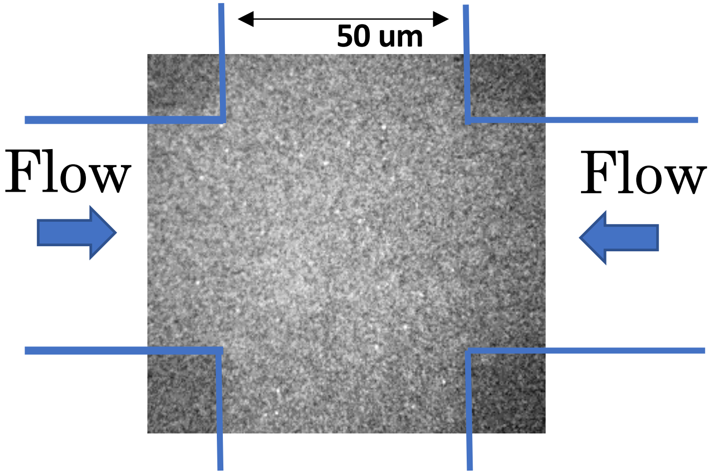


"hoop stresses"

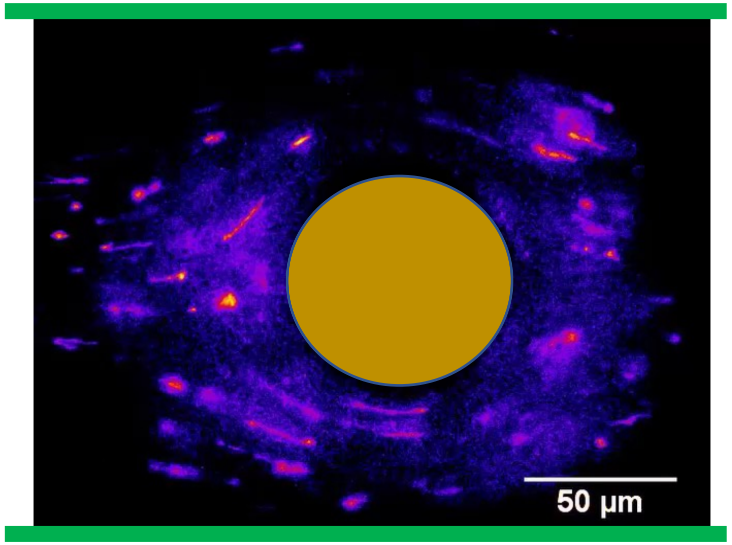
Polymer stretching by flow gradients



Perkins *et al*, *Science*, 1997



Arratia group



Arratia group

Purely elastic instabilities

Newtonian fluids

Viscoelastic fluids

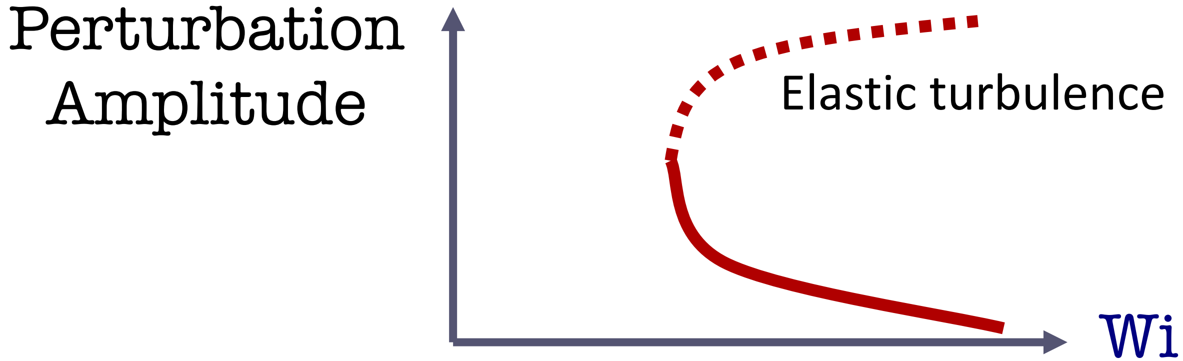
$$\text{Re} = \frac{UL}{\nu} \ll 1$$

but the anisotropic elastic forces (first normal stress difference) similarly tend to give instabilities

Larson; Groisman and Steinberg, Nature (2000)

Elastic instabilities in parallel shear flows: conjecture

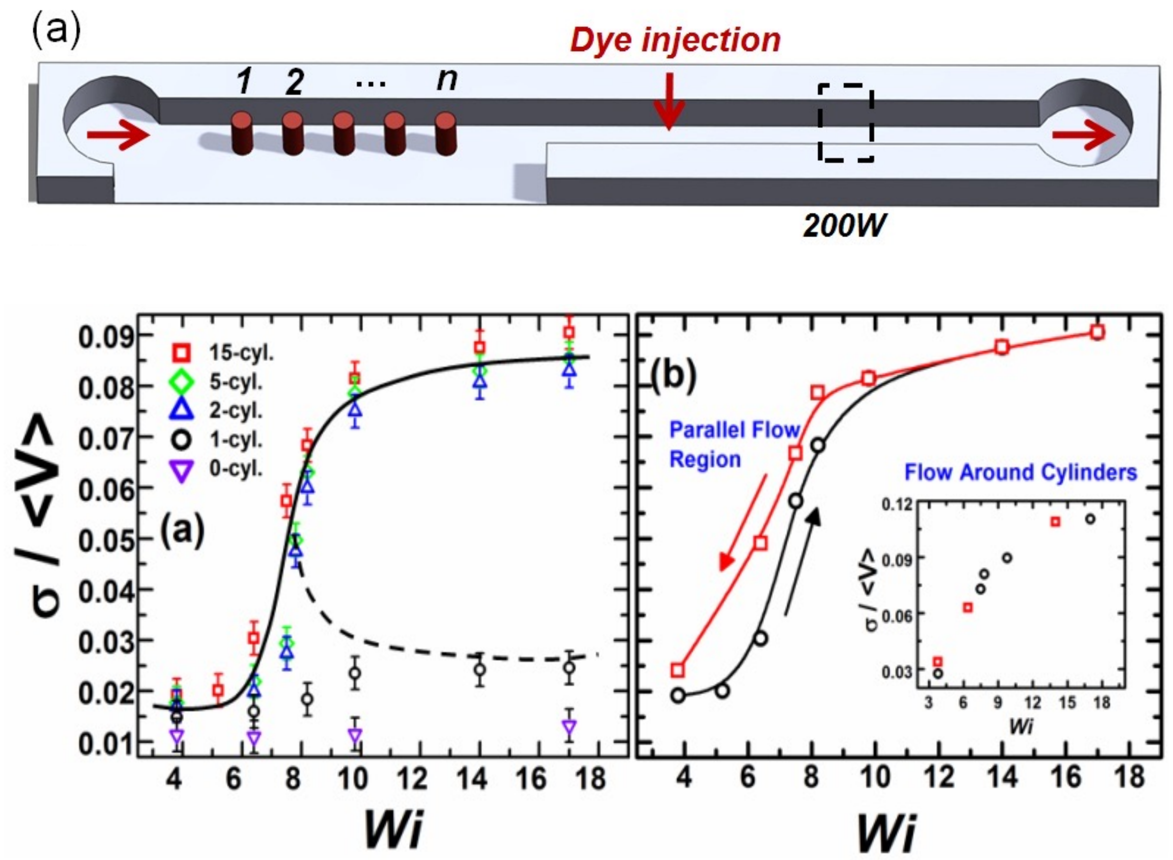
- Parallel streamlines → no linear instability
- Even though viscoelastic parallel shear flows a linearly stable, there could exist a finite-amplitude instability – similar to Newtonian turbulence



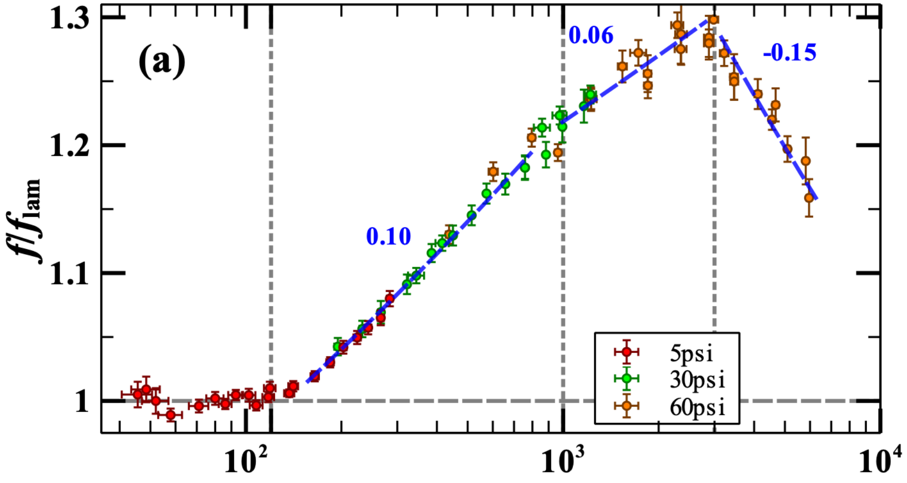
bifurcation from infinity
(sub-critical)

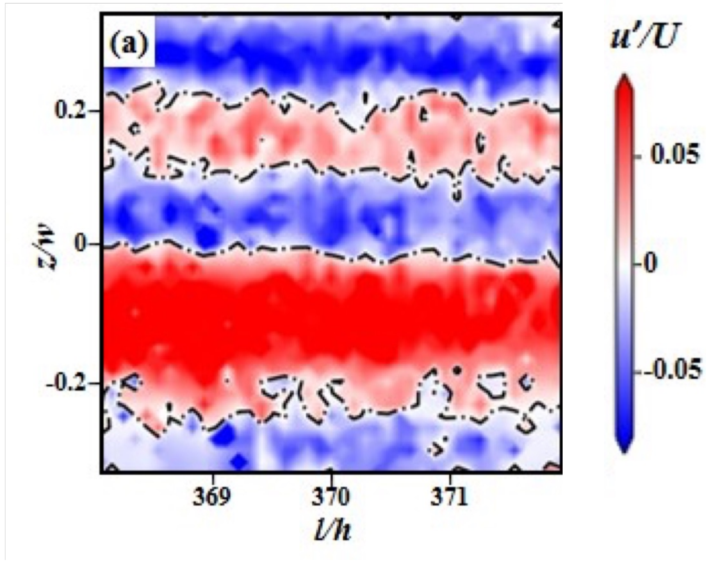
Bertola *et al.*, Phys. Rev. Lett. (2003)
AM and W. van Saarloos, Physics Reports (2007)

ET in parallel shear flows: experimental evidence



L. Pan *et al.*, *PRL (2013)*





Li & Steinberg arXiv:2201.06342 (2022)

Current state-of-the art summary:

- Sujit S. Datta, Arezoo M. Ardekani, Paulo E. Arratia, Antony N. Beris, Irmgard Bischofberger, Jens G. Eggers, J. Esteban López-Aguilar, Suzanne M. Fielding, Anna Frishman, Michael D. Graham, Jeffrey S. Guasto, Simon J. Haward, Sarah Hormozi, Gareth H. McKinley, Robert J. Poole, Alexander Morozov, V. Shankar, Eric S.G. Shaqfeh, Amy Q. Shen, Holger Stark, Victor Steinberg, Ganesh Subramanian, and Howard A. Stone. Perspectives on viscoelastic flow instabilities and elastic turbulence, *Phys. Rev. Fluids* **7,** 080701 (2022)

- Hugo A. Castillo Sanchez, Mihailo R. Jovanovic, Satish Kumar, Alexander Morozov, V. Shankar, Ganesh Subramanian, and Helen J. Wilson Understanding viscoelastic flow instabilities using the Oldroyd-B model, *J. Non-Newt. Fluid. Mech.* **302**, 104742 (2022)

2D coherent structures

AM Phys. Rev. Lett. **129**, 017801 (2022)

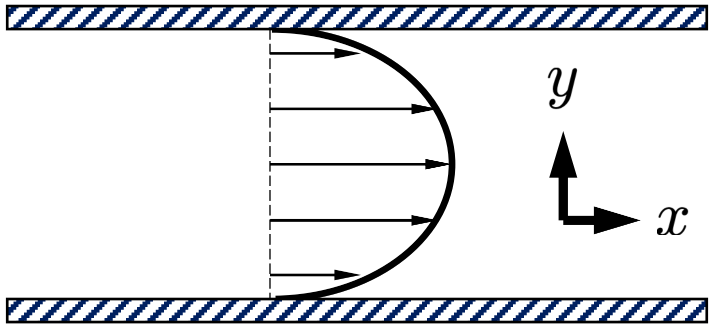
Flow Geometry and Equations of Motion

- 2D plane Poiseuille (channel) flow
- sPTT constitutive model

$$\frac{\partial \boldsymbol{c}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{c} - (\nabla \boldsymbol{v})^T \cdot \boldsymbol{c} - \boldsymbol{c} \cdot (\nabla \boldsymbol{v}) = \kappa \nabla^2 \boldsymbol{c} - \frac{\boldsymbol{c} - \mathbb{I}}{Wi} \left[1 - 2\,\epsilon + \epsilon \, \mathrm{Tr} \boldsymbol{c} \right]$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\frac{1}{Re} \nabla p + \frac{\beta}{Re} \nabla^2 \boldsymbol{v} + \frac{(1-\beta)}{Re\, Wi} \nabla \cdot \boldsymbol{c} + \begin{pmatrix} 2/Re \\ 0 \end{pmatrix}$$

$$\nabla \cdot \boldsymbol{v} = 0 \qquad \& \qquad \boldsymbol{v}(x, y = \pm 1, t) = 0$$

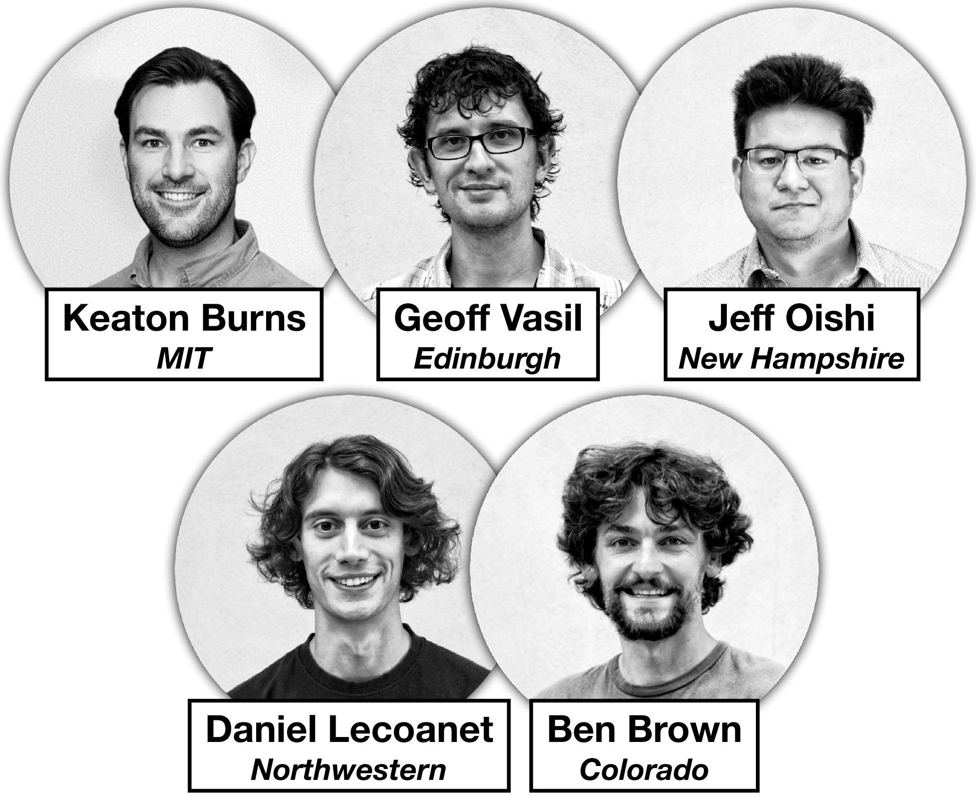


- Dimensionless groups

$$Wi = \frac{\lambda U}{d} \qquad Re = \frac{\rho\, U d}{\eta_s + \eta_p} \qquad \beta = \frac{\eta_s}{\eta_s + \eta_p}$$

- Conformation tensor boundary conditions

DEDALUS PROJECT



Keaton Burns
MIT

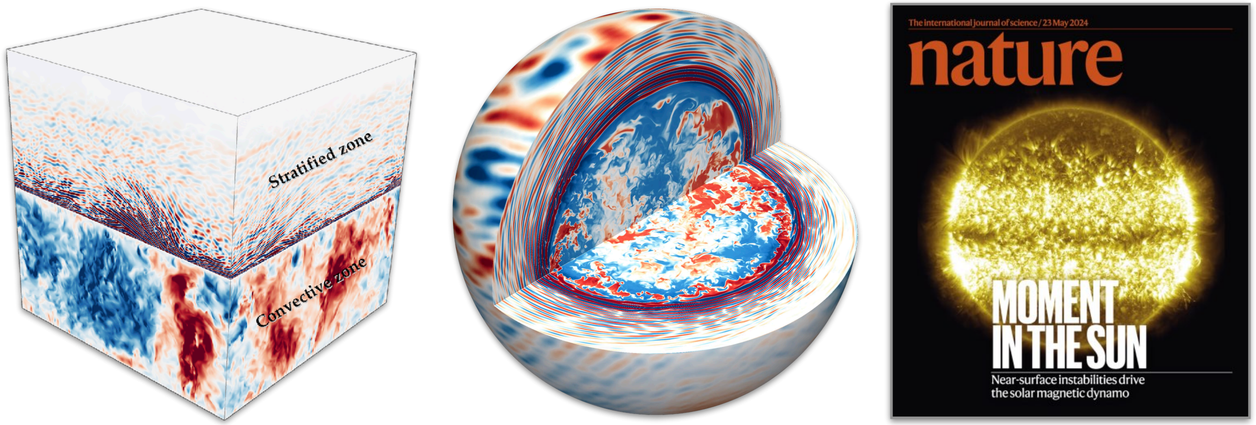
Geoff Vasil
Edinburgh

Jeff Oishi
New Hampshire

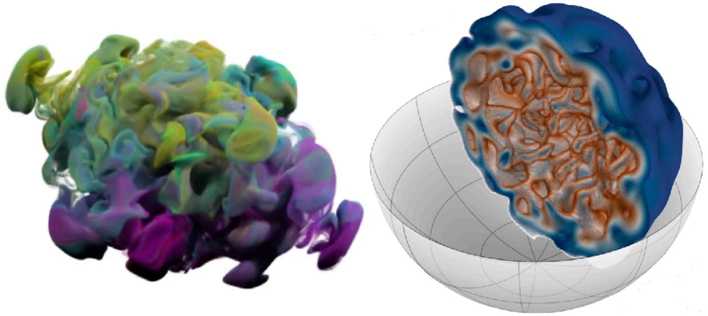
Daniel Lecoanet
Northwestern

Ben Brown
Colorado

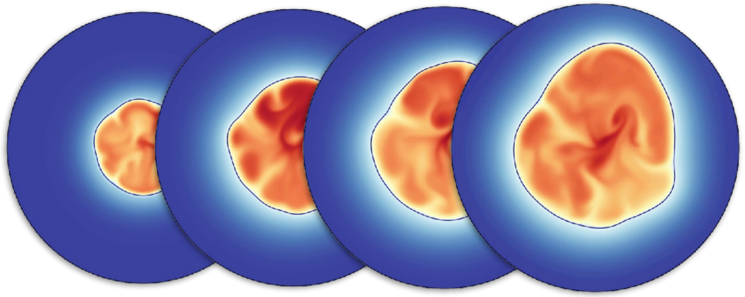
Global spectral methods for Cartesian & curvilinear domains



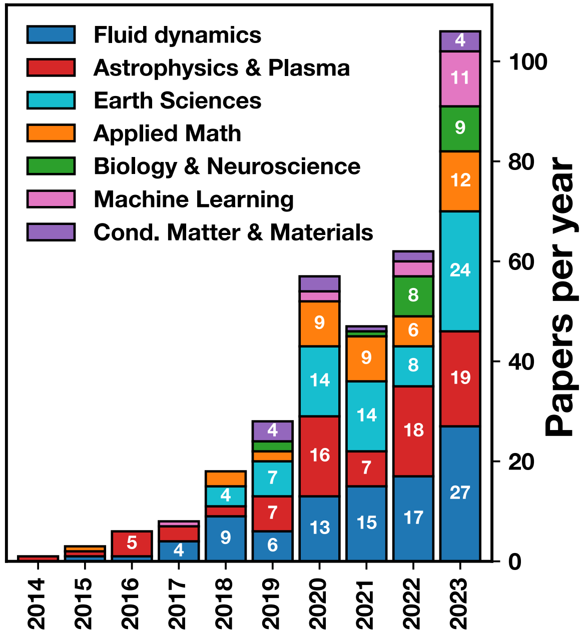
Flexible model development



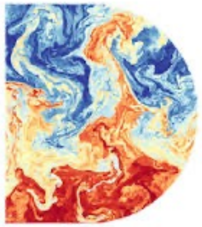
Phase-fields & immersed boundaries



Interdisciplinary applications

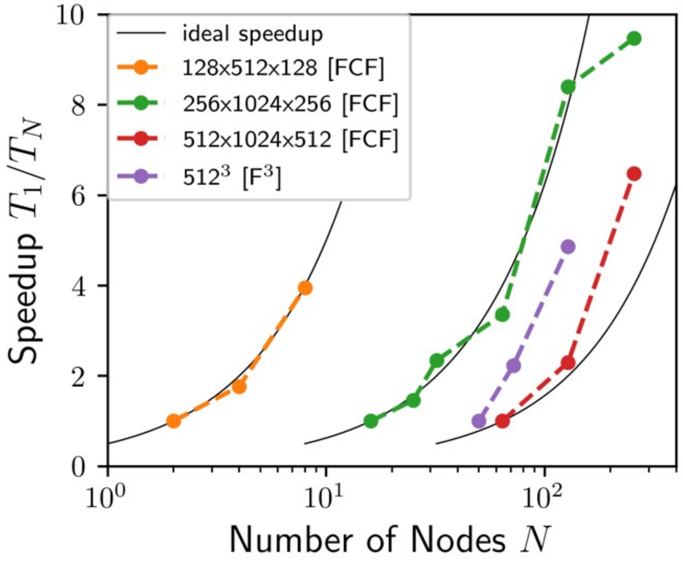


Direct Numerical Simulations



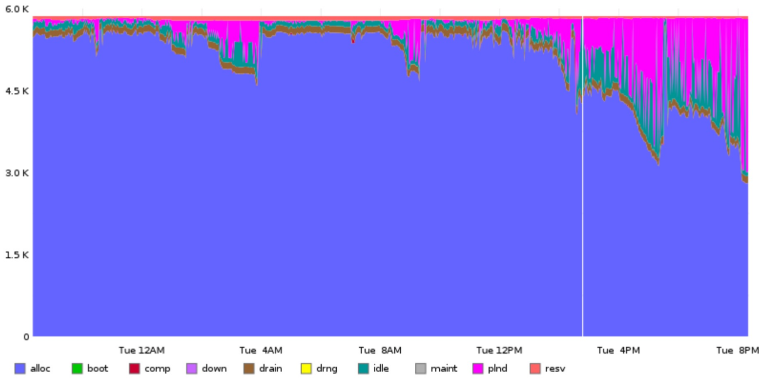
- Spectral in-house code implemented in Dedalus

K. Burns et al. Phys. Rev. Res. **2**, 023068 (2020)



- Fourier & Chebyshev discretization (256 x 1024 modes)
- RK443 time stepping $dt = 5 \cdot 10^{-3}$
- Parameters:

$$L_x = 10 \quad Re = 10^{-2} \quad \epsilon = 10^{-3} \quad \kappa = 5 \cdot 10^{-5}$$



- Initial conditions:

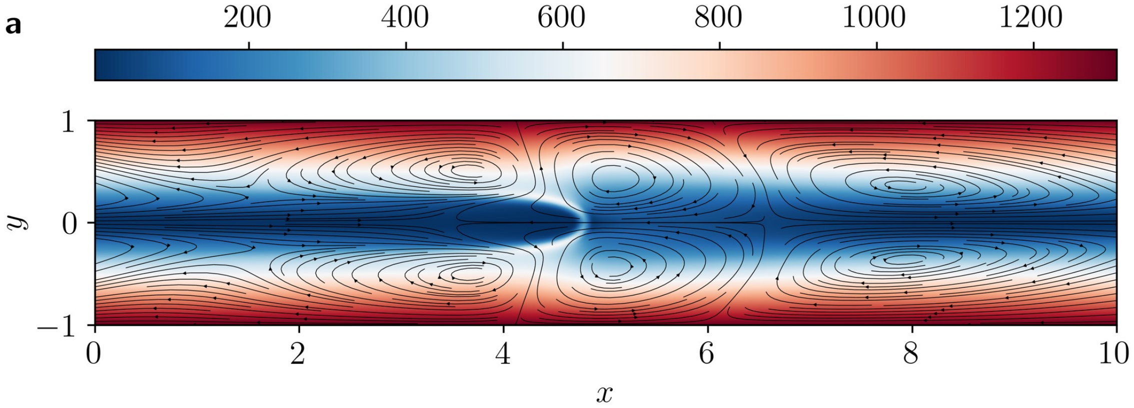
$$c_{xx}(x,y,t=0) = \Delta \exp\left[-\frac{25}{8}\left\{\left(\frac{2x}{L_x}-1\right)^2 + y^2\right\}\right],$$

J. Schumacher & B. Eckhardt, Phys. Rev. E **63**, 046307 (2001)

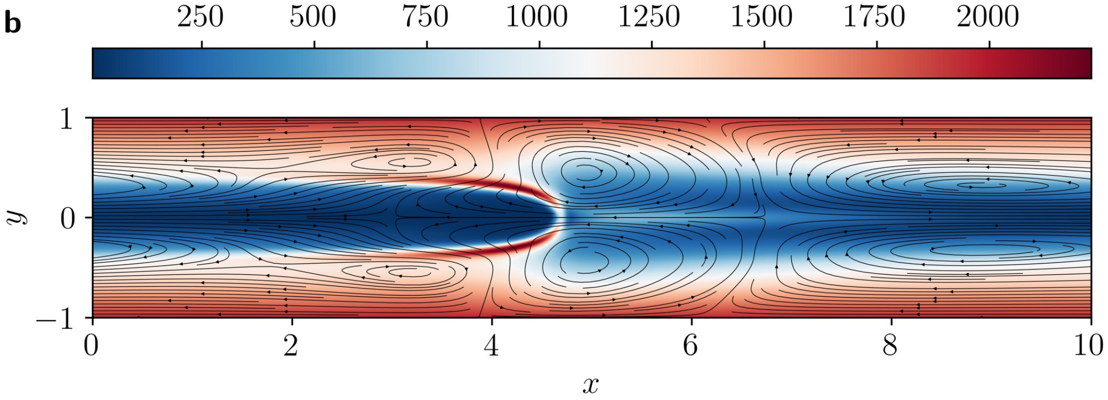
- Integrate until: $\dfrac{dE}{dt} < 10^{-10}$

Travelling wave solutions → Narwhal* states

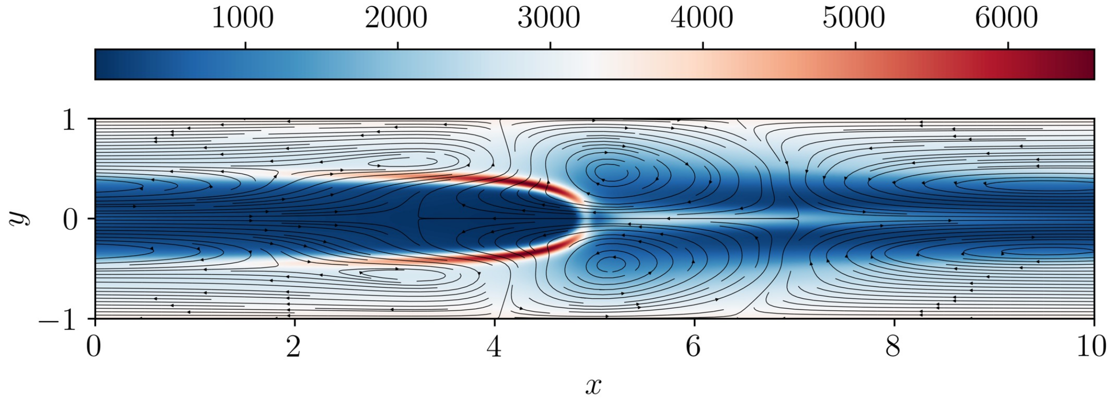
$Wi = 26$



$Wi = 40$



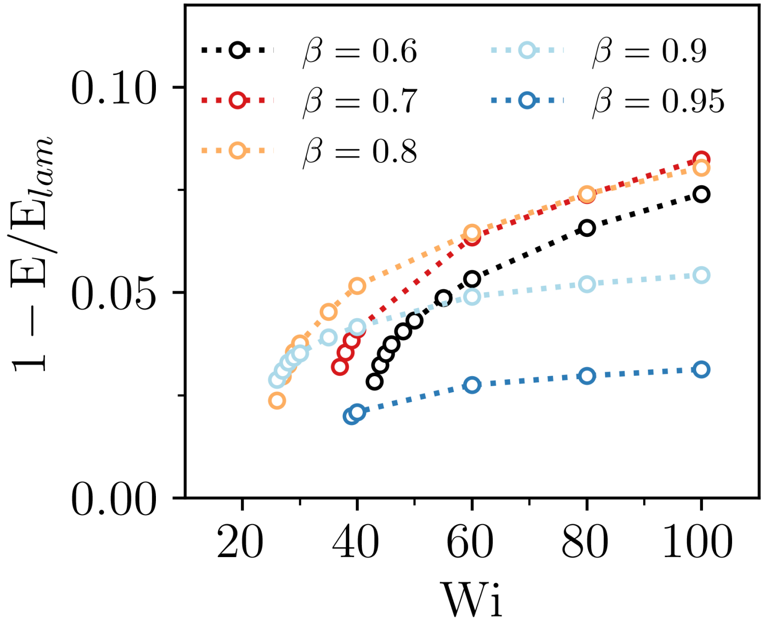
$Wi = 80$

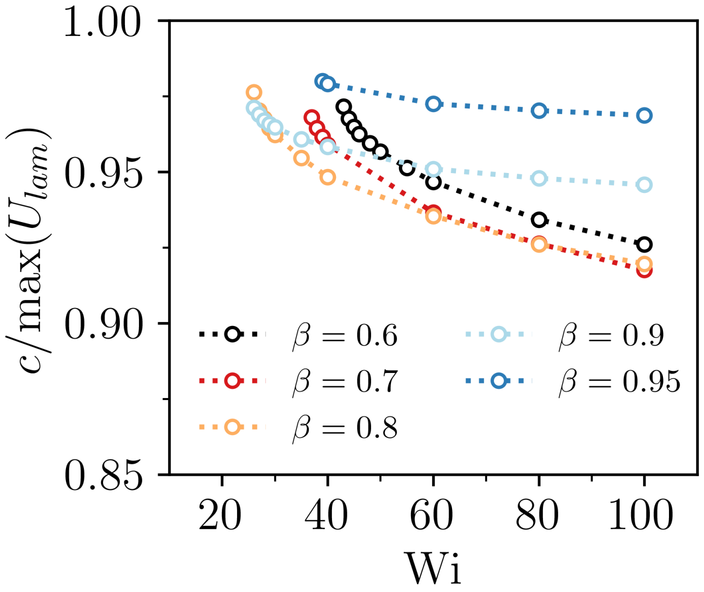


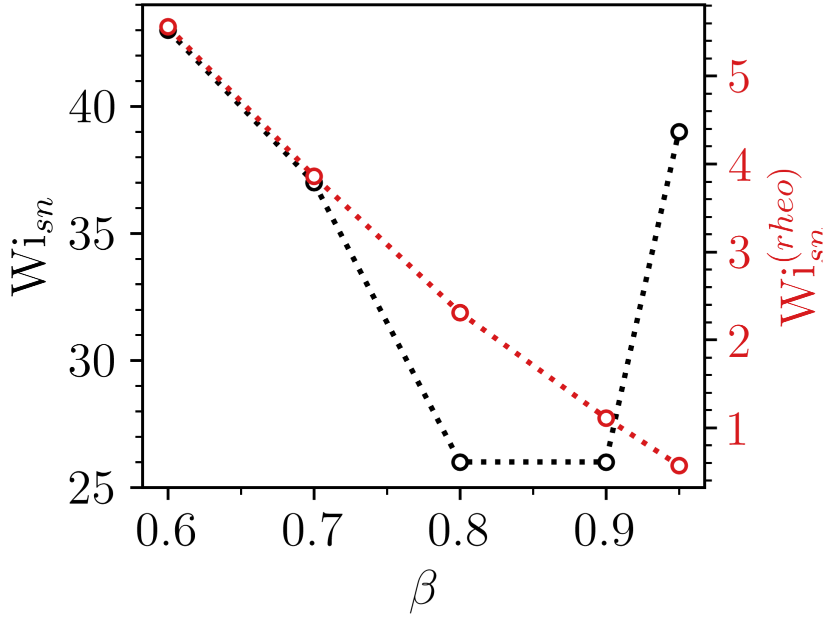
$\beta = 0.8$

*Arrowhead structures first found in EIT (Page et al. PRL **125**, 154501 (2020))

Narwhal states





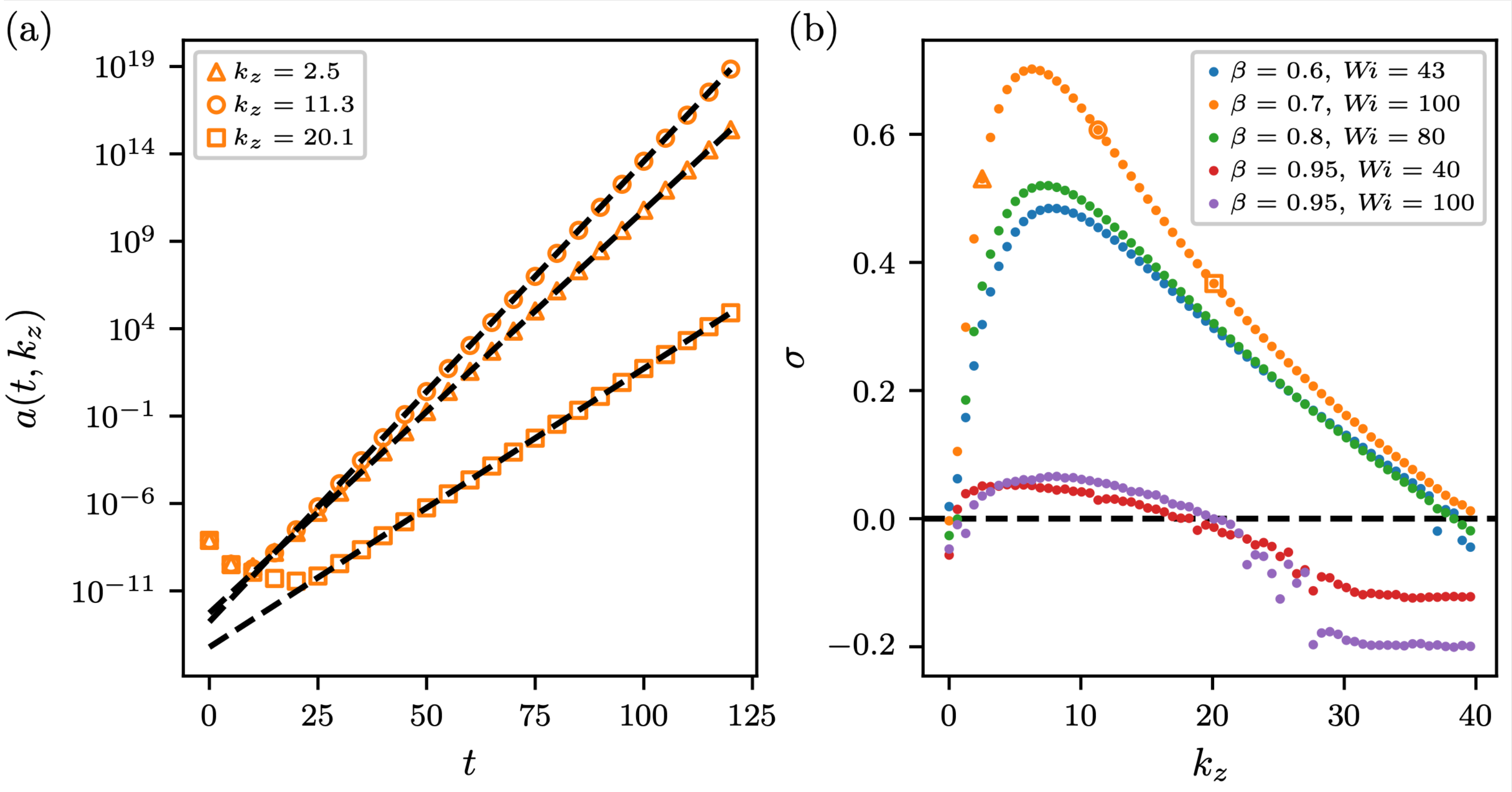


$$Wi^{(rheo)}(Wi) = \frac{N_1(Wi)}{2\tau_{xy}(Wi)}$$

L. Pan et al., Phys. Rev. Lett. **110**, 174502 (2013): $Wi_{sn}^{(rheo)}(Wi) \approx 6$ $\beta \approx 0.4$

3D linear stability analysis of the Narwhal states

Lellep et al. J. Fluid Mech. **959**, R1 (2023)



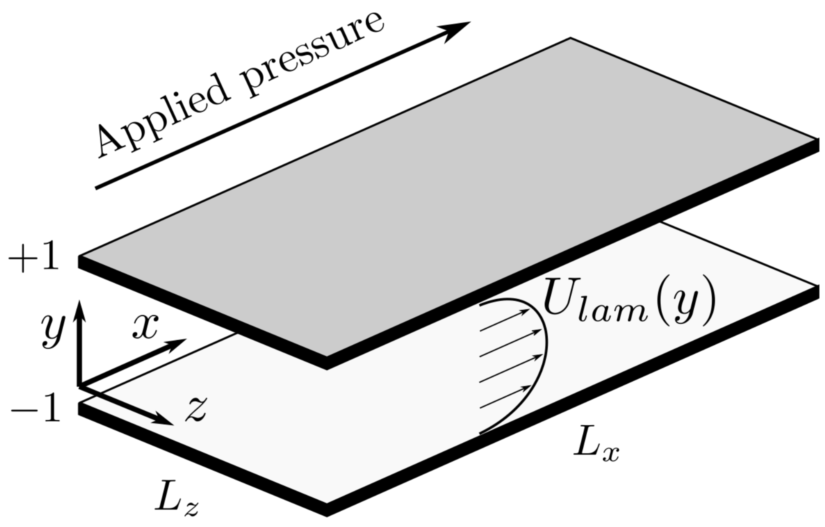
Conclusion: <u>all</u> 2D states are linearly unstable in 3D

3D structures & elastic turbulence

M. Lellep, M. Linkmann, AM (PNAS, **121**, e2318851121 (2024))

Dedalus 2 code available:

Governing Equations and Parameters



- 3D pressure-driven plane Poiseuille (channel) flow
- PTT constitutive model
- Fourier-Fourier & Chebyshev discretization

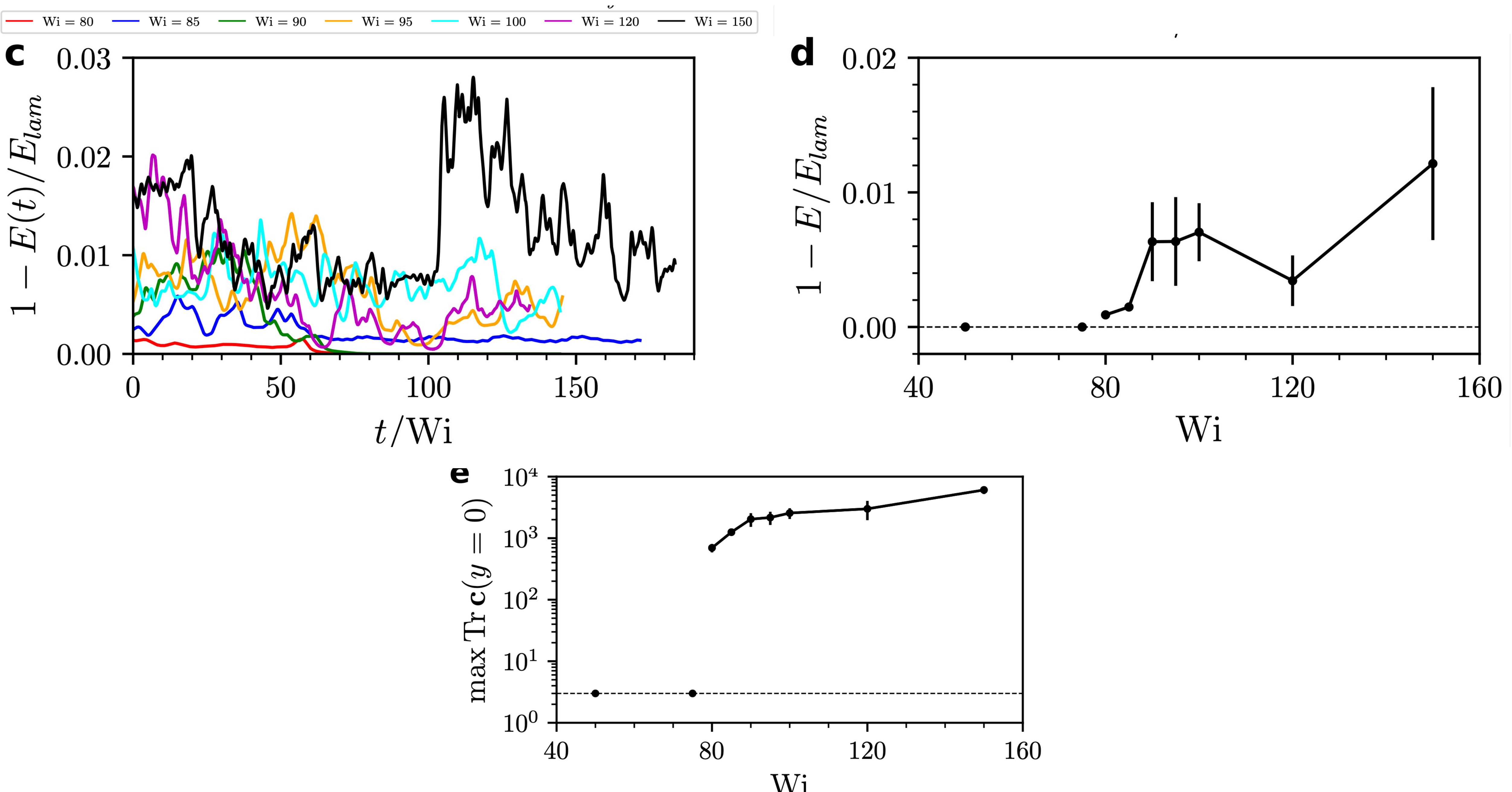
  (256 x 256 x 1024 or 512 x 512 x 2048 modes)
- SBDF4 time stepping $dt = 5 \cdot 10^{-3}$
- Parameters:

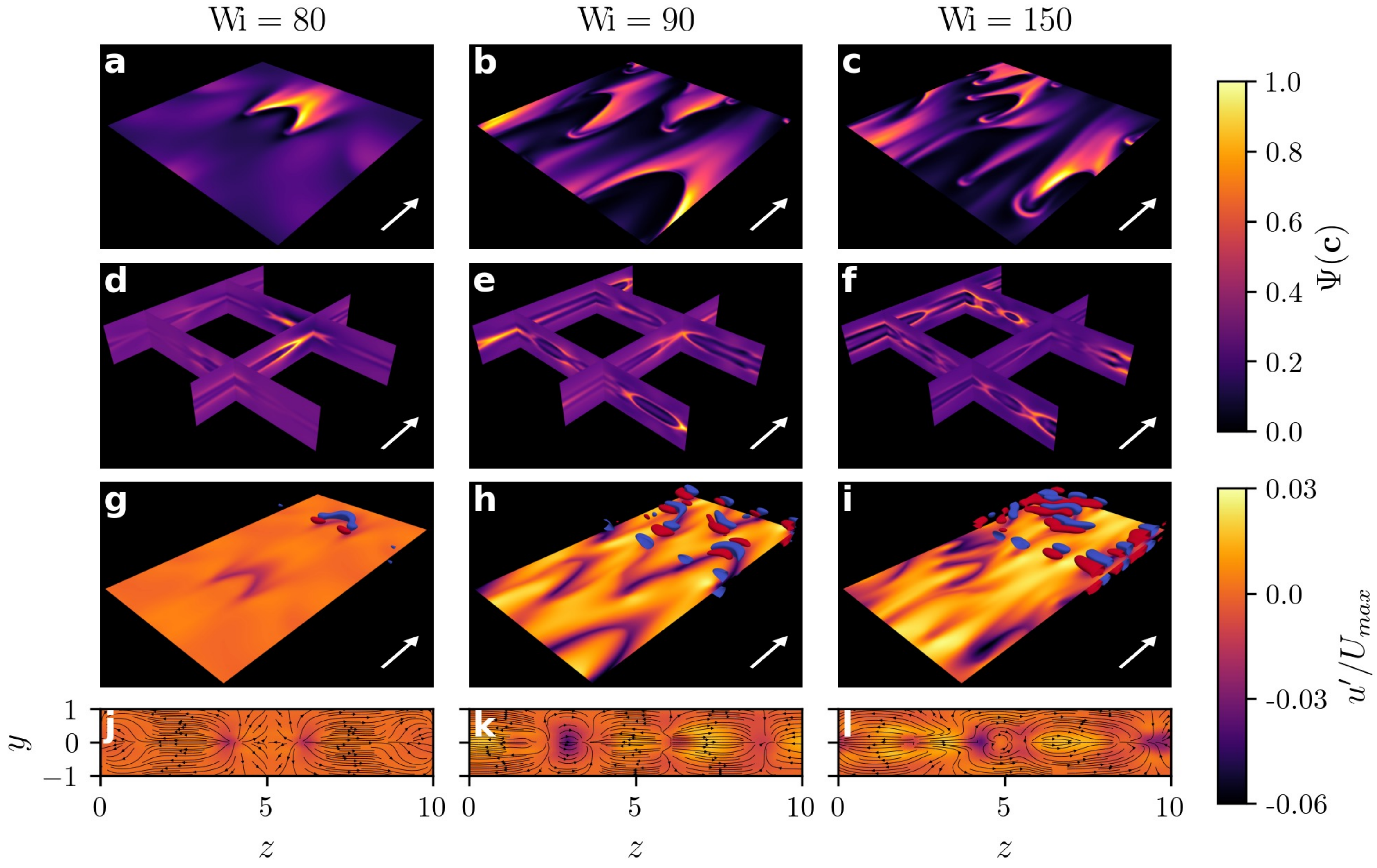
$L_x = 10$ $Re = 10^{-2}$ $\epsilon = 10^{-3}$ $\kappa = 5 \cdot 10^{-5}$

$L_z = 10$ $\beta = 0.8$

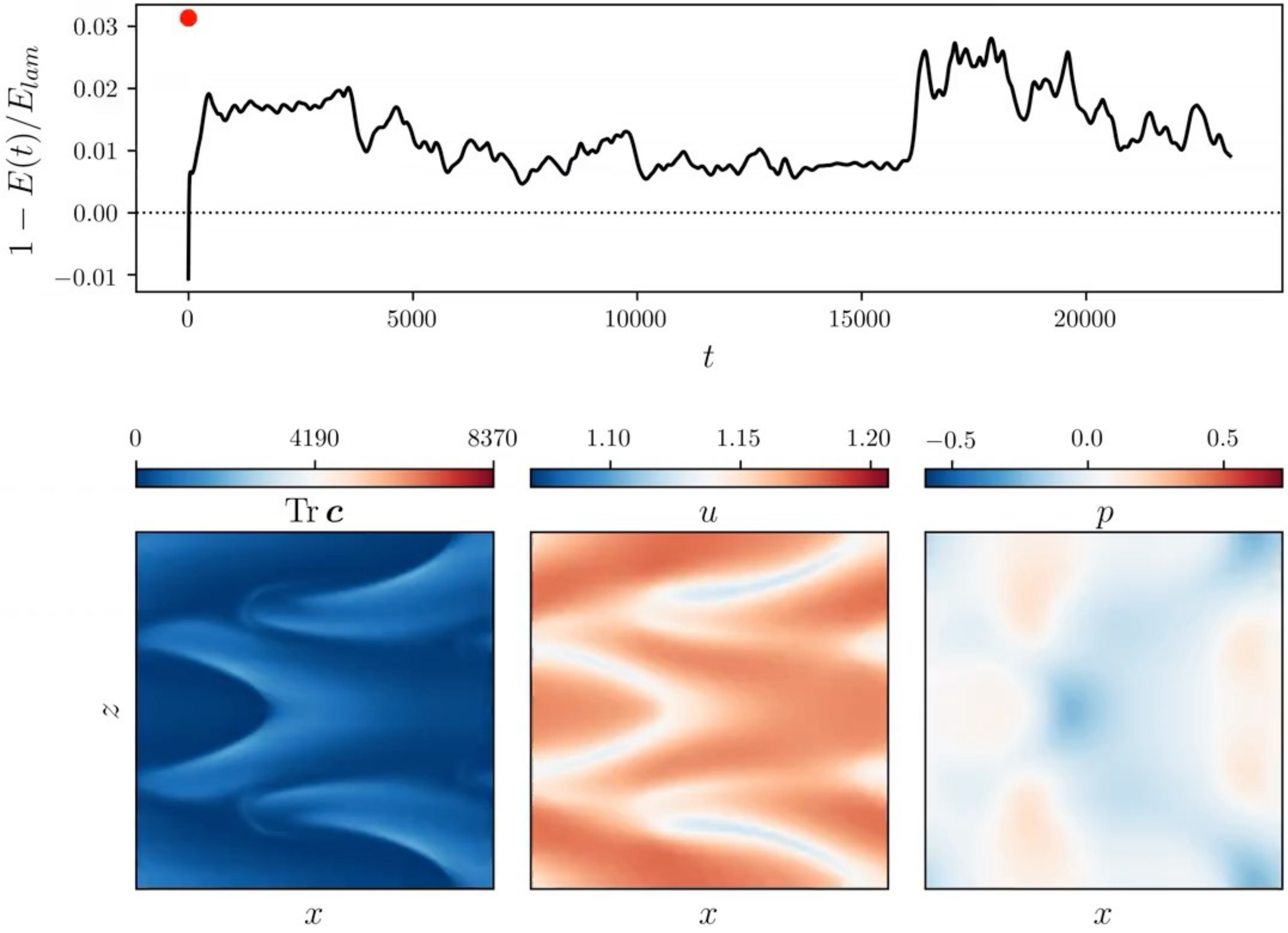
- About 10^9-10^{10} degrees of freedom

Bifurcation Diagram

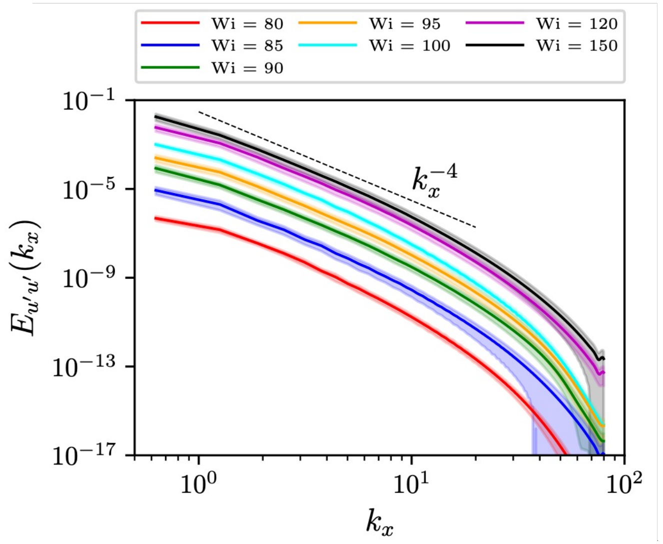


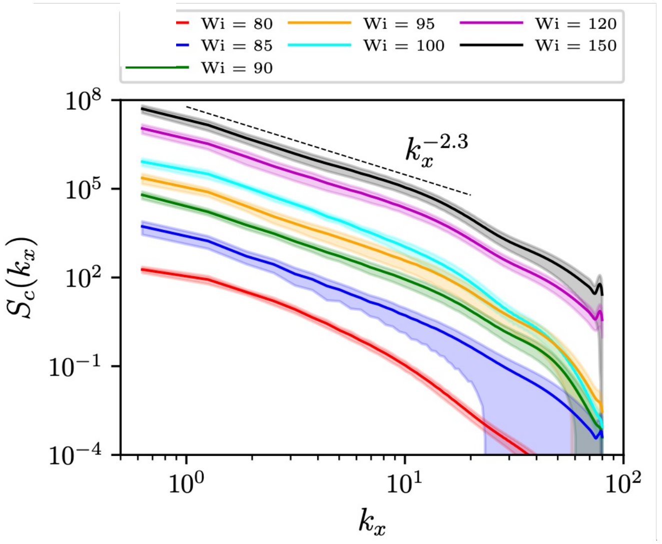

Wi = 80
Wi = 90
Wi = 150
a
b
c
d
e
f
g
h
i
j
k
l
1.0
0.8
0.6
0.4
0.2
0.0
$\Psi(\mathbf{c})$
0.03
0.0
-0.03
-0.06
u'/U_{max}
y
1
0
−1
0
5
10
z

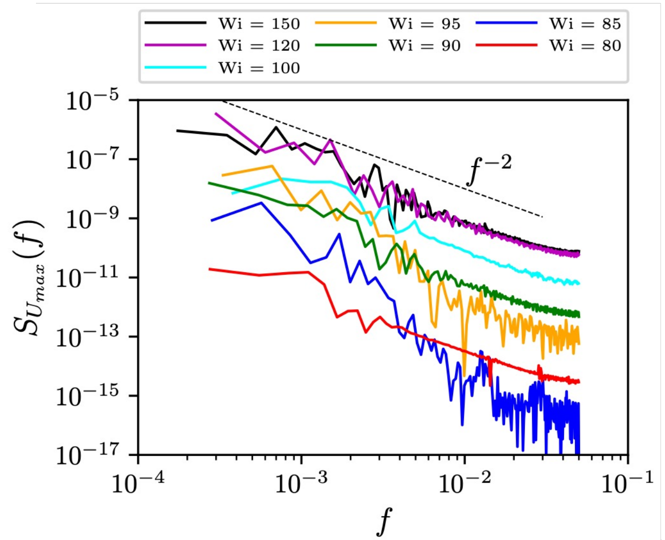
In-plane video $\mathrm{Wi} = 150$

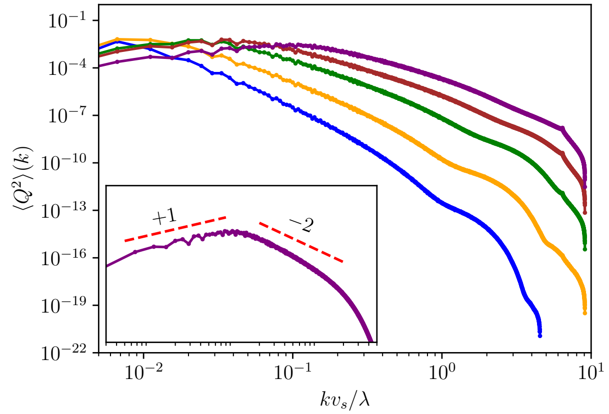


Turbulent spectra

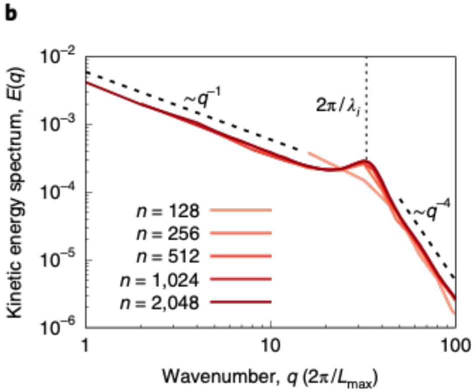




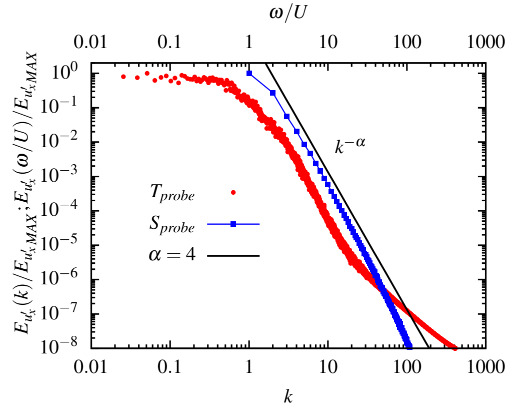




Negro et al.



Alert et al. Nat. Phys. **16**, 682–688 (2020)

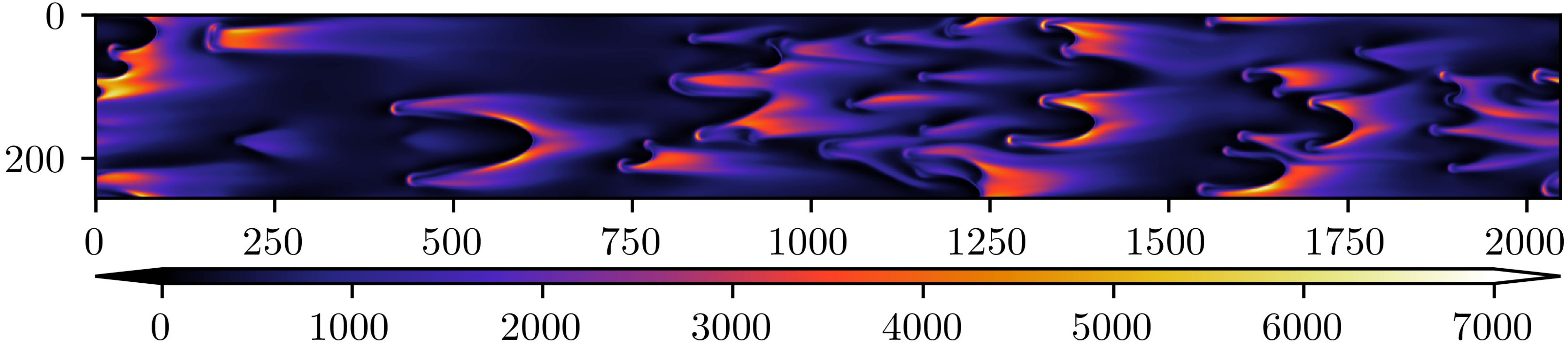


Garg et al. arXiv:2104.08951

Narwhal blessings in long domains

$\beta = 0.8 \quad Re = 10^{-2} \quad \kappa = 5 \cdot 10^{-5} \quad Wi = 150 \quad \epsilon = 10^{-3}$

$L_x \times L_y \times L_z = 80 \times 2 \times 10 \quad N_x \times N_y \times N_z = 2048 \times 128 \times 256$



Conclusions:

arXiv:2509.03175

- Elastic turbulence is very different from its Newtonian counterpart
- Bifurcation from infinity in purely elastic channel flow
- Coherent structures: unstable travelling wave solutions

Where to from here?

- Diversify (LUMI-C?)
- CPU → GPU transition; next eCSE call??